# Congruence and Transformations

Explore You can compare figures to determine if they are the same size and shape.

STEP 1 Copy the figure shown on tracing paper two times. Cut out both figures. Label the figures A and B.

STEP 2 Place Figure B on top of Figure A. Are the figures the same?

STEP 3 Slide Figure B up and over on your desk. Can you move Figure A on top of Figure B so all sides and angles match?

**STEP 4** Flip Figure B over. Can you move Figure A on top of Figure B so all sides and angles match?
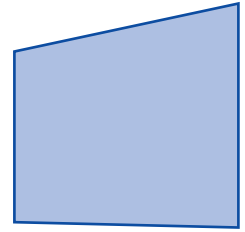
In the activity above, you matched Figure A to Figure B by a translation and a reflection. Two figures are congruent if the second can be obtained from the first by a series of rotations, reflections, and/or translations.
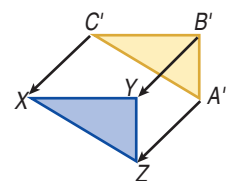
EXAMPLE **Identify Congruency**

1 **Determine if the two figures are congruent by using transformations.**
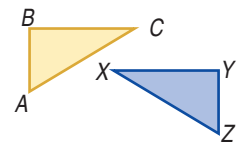
Step 1 Reflect $\triangle ABC$ over a vertical line. Label the vertices of the image A', B', and C'.
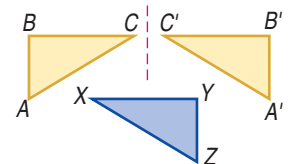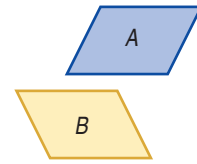
Step 2 Translate $\triangle A'B'C'$ until all sides and angles match $\triangle XYZ$.

So, the two triangles are congruent because a reflection followed by a translation will map $\triangle ABC$ onto $\triangle XYZ$.

a. Determine if the two figures are congruent by using transformations. Explain your reasoning.

EXAMPLE

2 **Determine if the two figures are congruent by using transformations.**

Reflect the red figure over a vertical line.

Even if the reflected figure is translated up and over, it will not match the green figure exactly. The two figures are not congruent.

✓ **CHECK Your Progress**

b. Determine if the two figures are congruent by using transformations. Explain your reasoning.

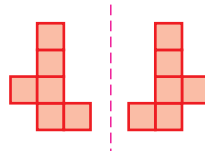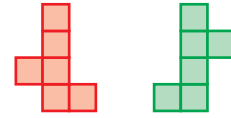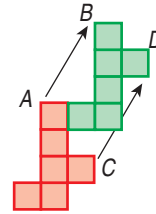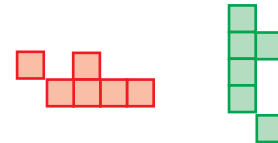If you have two congruent figures, you can determine the transformation, or series of transformations, that maps one figure onto the other by analyzing the orientation or relative position of the figures.

Translation	Reflection	Rotation
• length is the same • orientation is the same	• length is the same • orientation is reversed	• length is the same • orientation is changed
Notice the segments are facing the same way.	Notice the segments are facing the opposite way.	Notice the segments are facing a different way.

3 **GRAPHIC DESIGN** Ms. Martinez created the logo shown. What transformations did she use if the letter "d" is the preimage and the letter "p" is the image? Are the two figures congruent?

Step 1 Start with the preimage. Determine which transformation will change the orientation of the letter.

Step 2 Rotations or reflections change orientation. Rotate the letter "d" 180° about point *A*.

Step 3 Translate the new image up.

So, Ms. Martinez used a rotation and translation to create the logo. The letters are congruent because images produced by a rotation and translation have the same shape and size.

Check Trace the letter "d" with tracing paper. Rotate the letter 180° around Point *A*. Slide it up to line up with the letter "p." The letters are the same shape and size. They are congruent ✔
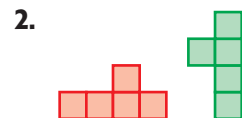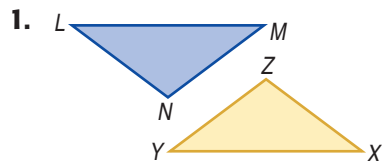
✓ **CHECK Your Progress**

c. GRAPHIC DESIGN What transformations could be used if the letter "W" is the preimage and the letter "M" is the image in the logo shown?
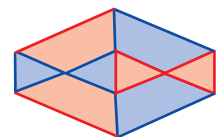

CHECK Your Understanding

Examples 1 and 2 Determine if the two figures are congruent by using transformations. Explain your reasoning.
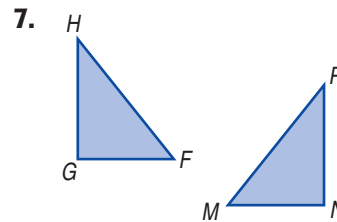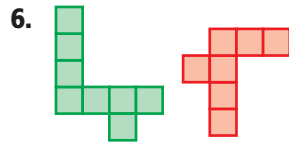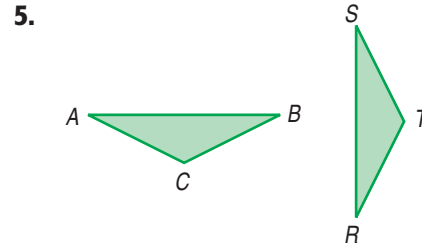
1. *L* *M* *N* *Z* *Y* *X*

2.

Example 3 **3. LOGOS** The Boyd Box Company uses the logo shown. What transformations could be used if the top, red trapezoid is the preimage and the bottom, blue trapezoid is the image?

Examples 1 and 2 Determine if the two figures are congruent by using transformations. Explain your reasoning.
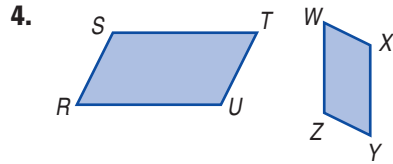
4.

5.

6.

7.

Example 3 **8. STATIONERY** Nilda purchased some custom printed stationery with her initials. What transformations could be used if the letter "Z" is the preimage and the letter "N" is the image in the design shown?

9. ART Simon is illustrating a graphic novel for a friend. He is using the two thought bubbles shown. What transformations did he use if Figure A is the preimage and Figure B is the image?
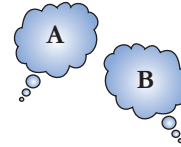
10. One way to identify congruent triangles is to prove their matching sides have the same measure. Triangle CDE has vertices at $(1, 4)$, $(1, 1)$, and $(5, 1)$.

   a. Find the lengths of the sides of $\triangle CDE$.

   b. Reflect $\triangle CDE$ over the y-axis, then translate it 2 units left. Label the vertices of the image $C'D'E'$.

   c. Find the lengths of the sides of $\triangle C'D'E'$.

   d. Are the two triangles congruent? Justify your response.
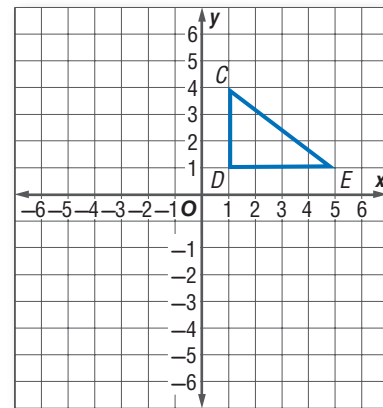
Find the lengths of the sides of the preimage with the given vertices and the image after the transformations are performed. Then determine if the two figures are congruent.

11. preimage: $(0, 1)$, $(4, 0)$, $(4, 1)$
   transformations: translate 3 units up then reflect over the y-axis

12. preimage: $(0, 0)$, $(4, 0)$, $(0, 4)$
   transformations: reflect over the x-axis then dilate by a scale factor of 2

**13. OPEN ENDED** Create a design using a series of transformations that produce congruent figures. Exchange designs with a classmate and determine what transformations were used to create their design.

14. CHALLENGE Angle *ABC* has points *A*(−3, 4), *B*(−2, 1) and *C*(2, 2). Find the coordinates of the image of the angle after a 90° clockwise rotation about the origin, a translation of 2 units up, and a reflection over the *y*-axis.

15. CHALLENGE Line segment *XY* has endpoints at *X*(3, 1) and *Y*(−2, 0). Its image after a series of transformations has endpoints at *X'*(0, 1) and *Y'*(5, 0). Find the series of transformations that maps $\overline{XY}$ onto $\overline{X'Y'}$. Then find the exact length of both segments.

16. WRITE MATH Explain why rotations, reflections, and translations create congruent images.
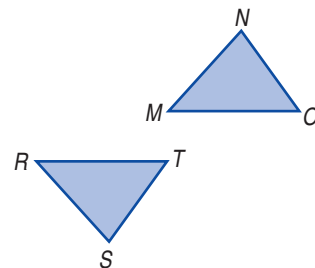
Test Practice

17. SHORT RESPONSE Gregory is creating a mosaic for art class. He started by using triangular tiles as shown.
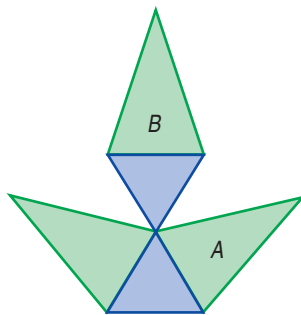
What are possible transformations he used if Figure A is the preimage and Figure B is the image?

18. Triangle *MNO* is congruent to triangle *RST*.

Which series of transformations maps △*MNO* onto △*RST*?

A. 90° clockwise rotation about *M* then reflection

B. translation then dilation

C. 90° clockwise rotation about *M* then translation

D. reflection then translation