4.8 Perform Congruence Transformations

Goal • Create an image congruent to a given triangle.

Your Notes

VOCABULARY

Transformation A transformation is an operation that moves or changes a geometric figure in some way to produce a new figure.

Image The new figure produced by a transformation is the image.

Translation A translation moves every point of a figure the same distance in the same direction.

Reflection A reflection uses a *line of reflection* to create a mirror image of the original figure.

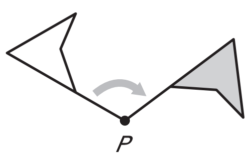
Rotation A rotation turns a figure about a fixed point, called the *center of rotation*.

Congruence Transformation A congruence transformation changes the position of a figure without changing its size or shape.

Example 1 ***Identify transformations***

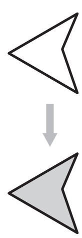
Name the type of transformation demonstrated in each picture.

a.



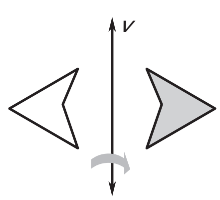
Rotation about a point

b.



Translation in a straight path

c.



Reflection in a vertical line

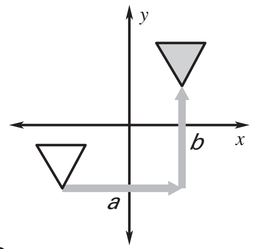
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$(x, y) \to (x + a, y + b)$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



Example 2 ***Translate a figure in the coordinate plane***

Figure $ABCD$ has the vertices $A(1, 2)$, $B(3, 3)$, $C(4, -1)$, and $D(1, -2)$. Sketch $ABCD$ and its image after the translation $(x, y) \to (x - 4, y + 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by <u>subtracting 4</u> from its x-coordinate and <u>adding 2</u> to its y-coordinate. Then draw $ABCD$ and its image.

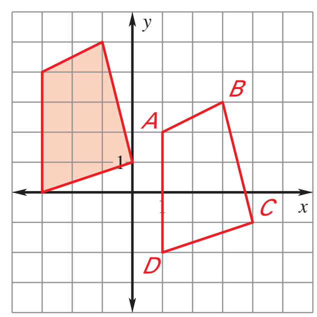
$(x, y) \to (x - 4, y + 2)$

$A(1, 2) \to$ <u>$(-3, 4)$</u>

$B(3, 3) \to$ <u>$(-1, 5)$</u>

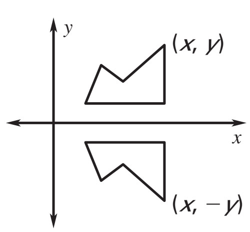
$C(4, -1) \to$ <u>$(0, 1)$</u>

$D(1, -2) \to$ <u>$(-3, 0)$</u>



COORDINATE NOTATION FOR A REFLECTION

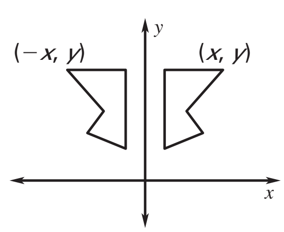
Reflection in the x-axis



Multiply y-coordinate by -1.

$(x, y) \to (x, -y)$

Reflection in the y-axis



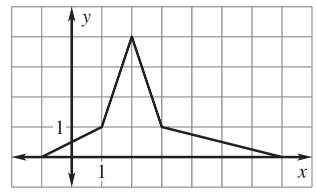
Multiply x-coordinate by -1.

$(x, y) \to (-x, y)$

Your Notes

Example 3 *Reflect a figure in the x-axis*

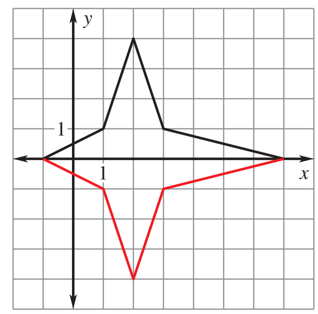
Shapes You are cutting figures out of paper. Use a reflection in the x-axis to draw the other half of the figure.



Solution

Multiply the y-coordinate of each vertex by -1 to find the corresponding vertex in the image. Then draw the image.

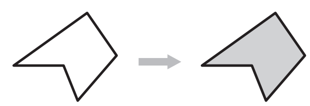
$(x, y) \rightarrow (x, -y)$
$(-1, 0) \rightarrow (-1, 0)$
$(1, 1) \rightarrow (1, -1)$
$(2, 4) \rightarrow (2, -4)$
$(3, 1) \rightarrow (3, -1)$
$(7, 0) \rightarrow (7, 0)$



You can check your results by looking to see if each original point and its image are the same distance from the x-axis.

Checkpoint Complete the following exercises.

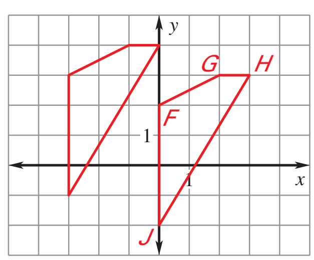
1. Name the type of transformation shown.



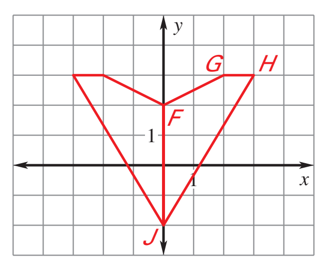
Translation

2. Figure $FGHJ$ has the vertices $F(0, 2)$, $G(2, 3)$, $H(3, 3)$, and $J(0, -2)$. Sketch $FGHJ$ and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the y-axis.

a.



b.



Your Notes

Example 4 *Identify a rotation*

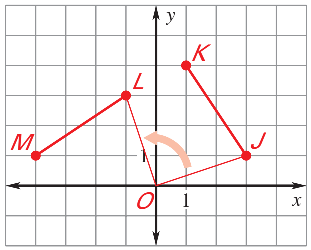
Graph $\overline{JK}$ and $\overline{LM}$. Tell whether $\overline{LM}$ is a rotation of $\overline{JK}$ about the origin. If so, give the angle and direction of rotation.

a. $J(3, 1)$, $K(1, 4)$, $L(-1, 3)$, $M(-4, 1)$

b. $J(-2, 1)$, $K(-1, 5)$, $L(1, 1)$, $M(2, 5)$

Solution

a.

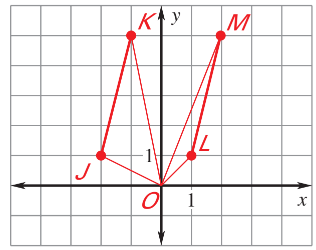


$m\angle JOL$ = $m\angle KOM$

= 90°

90° counterclockwise rotation

b.

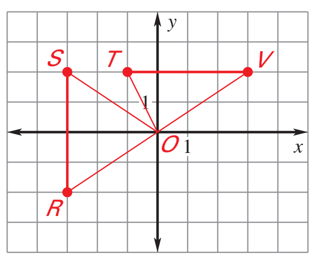


$m\angle JOL$ > $m\angle KOM$

not a rotation

Checkpoint Graph $\overline{RS}$ and $\overline{TV}$. Tell whether $\overline{TV}$ is a rotation of $\overline{RS}$ about the origin. If so, give the angle of rotation.

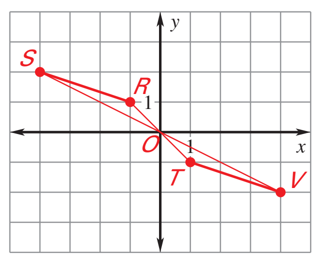
3. $R(-3, -2)$, $S(-3, 2)$, $T(-1, 2)$, $V(3, 2)$



$m\angle ROT < m\angle SOV$

not a rotation

4. $R(-1, 1)$, $S(-4, 2)$, $T(1, -1)$, $V(4, -2)$



$m\angle ROT = m\angle SOV$

$= 180°$

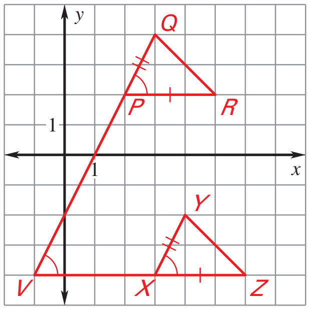
180° rotation

Your Notes

Example 5 ***Verify congruence***

The vertices of $\triangle PQR$ are $P(2, 2)$, $Q(3, 4)$, and $R(5, 2)$. The notation $(x, y) \to (x + 1, y - 6)$ describes the translation of $\triangle PQR$ to $\triangle XYZ$. Show that $\triangle PQR \cong \triangle XYZ$ to verify that the translation is a congruence transformation.

Solution



S You can see that $PR =$ XZ $=$ 3, so $\overline{PR} \cong$ $\overline{XZ}$.

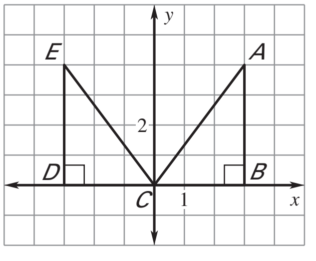
A Using the slopes, $\overline{PQ} \parallel$ $\overline{XY}$ and $\overline{QR} \parallel$ $\overline{YZ}$. If you extend $\overline{PQ}$ and $\overline{XZ}$ to form $\angle V$, the Corresponding Angles Postulate gives you $\angle QPR$ $\cong \angle V$ and $\angle V \cong$ $\angle YXZ$. Then, $\angle QPR$ $\cong$ $\angle YXZ$ by the Transitive Property of Congruence.

S Using the distance formula, $PQ =$ XY $=$ $\sqrt{5}$ so $\overline{PQ} \cong$ $\overline{XY}$. So, $\triangle PQR \cong \triangle XYZ$ by the SAS Congruence Postulate.

Because $\triangle PQR \cong \triangle XYZ$, the translation is a congruence transformation.

Checkpoint **Complete the following exercise.**

5. Show that $\triangle ABC \cong \triangle EDC$ to verify that the transformation is a congruence transformation.



You can see that $AB = ED = 4$ and $BC = DC = 3$. Also, $\angle B$ and $\angle D$ are congruent right angles. So, $\triangle ABC \cong \triangle EDC$ by the SAS Congruence Postulate.

Homework