

4.8

Perform Congruence Transformations

Goal • Create an image congruent to a given triangle.

Your Notes

VOCABULARY

Transformation

Image

Translation

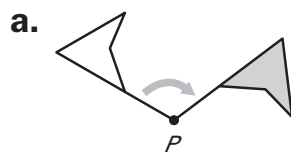
Reflection

Rotation

Congruence Transformation

Example 1 Identify transformations

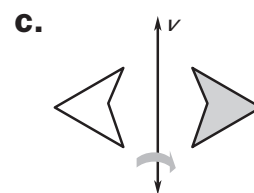
Name the type of transformation demonstrated in each picture.



_____ about a point



_____ in a straight path



_____ in a vertical line

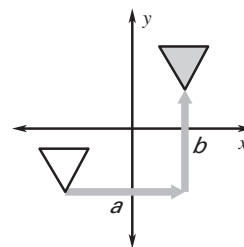
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$$(x, y) \rightarrow (x + a, y + b)$$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



Example 2 Translate a figure in the coordinate plane

Figure $ABCD$ has the vertices $A(1, 2)$, $B(3, 3)$, $C(4, -1)$, and $D(1, -2)$. Sketch $ABCD$ and its image after the translation $(x, y) \rightarrow (x - 4, y + 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by _____ from its x -coordinate and _____ to its y -coordinate. Then draw $ABCD$ and its image.

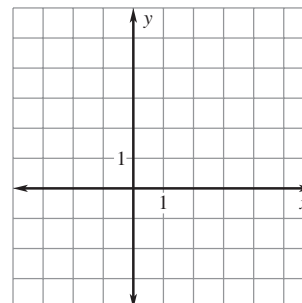
$$(x, y) \rightarrow (x - 4, y + 2)$$

$$A(1, 2) \rightarrow \underline{\hspace{2cm}}$$

$$B(3, 3) \rightarrow \underline{\hspace{2cm}}$$

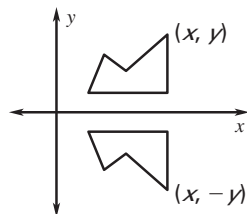
$$C(4, -1) \rightarrow \underline{\hspace{2cm}}$$

$$D(1, -2) \rightarrow \underline{\hspace{2cm}}$$



COORDINATE NOTATION FOR A REFLECTION

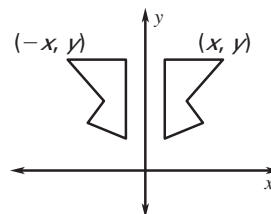
Reflection in the x -axis



Multiply y -coordinate by -1 .

$$(x, y) \rightarrow (x, -y)$$

Reflection in the y -axis



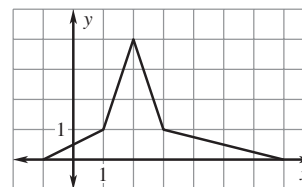
Multiply x -coordinate by -1 .

$$(x, y) \rightarrow (-x, y)$$

Your Notes

Example 3 Reflect a figure in the x-axis

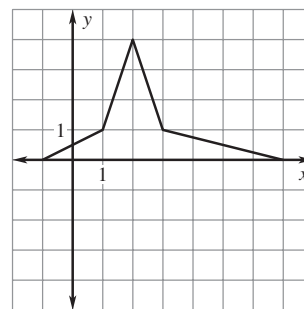
Shapes You are cutting figures out of paper. Use a reflection in the x-axis to draw the other half of the figure.



Solution

Multiply the _____ of each vertex by -1 to find the corresponding vertex in the image. Then draw the image.

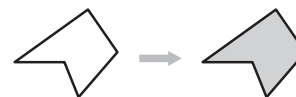
- $(x, y) \rightarrow$ _____
 $(-1, 0) \rightarrow$ _____
 $(1, 1) \rightarrow$ _____
 $(2, 4) \rightarrow$ _____
 $(3, 1) \rightarrow$ _____
 $(7, 0) \rightarrow$ _____



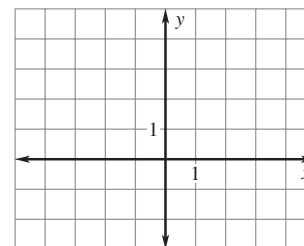
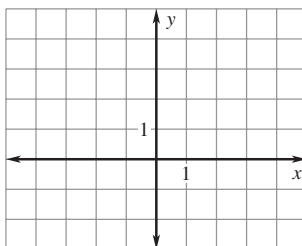
You can check your results by looking to see if each original point and its image are the same distance from the _____.

✓ Checkpoint Complete the following exercises.

1. Name the type of transformation shown.



2. Figure $FGHJ$ has the vertices $F(0, 2)$, $G(2, 3)$, $H(3, 3)$, and $J(0, -2)$. Sketch $FGHJ$ and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the y-axis.



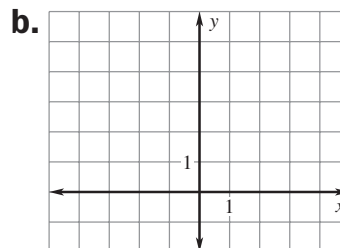
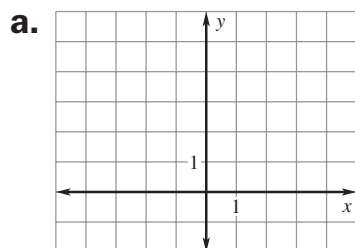
Example 4 Identify a rotation

Graph \overline{JK} and \overline{LM} . Tell whether \overline{LM} is a rotation of \overline{JK} about the origin. If so, give the angle and direction of rotation.

a. $J(3, 1), K(1, 4), L(-1, 3), M(-4, 1)$

b. $J(-2, 1), K(-1, 5), L(1, 1), M(2, 5)$

Solution

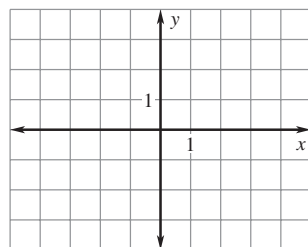


$m\angle JOL$ ____ $m\angle KOM$

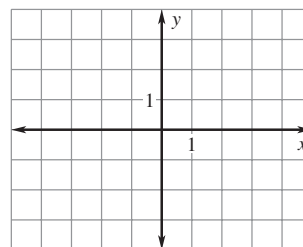
$m\angle JOL$ ____ $m\angle KOM$

Checkpoint Graph \overline{RS} and \overline{TV} . Tell whether \overline{TV} is a rotation of \overline{RS} about the origin. If so, give the angle of rotation.

3. $R(-3, -2), S(-3, 2), T(-1, 2), V(3, 2)$



4. $R(-1, 1), S(-4, 2), T(1, -1), V(4, -2)$



Your Notes

Example 5 *Verify congruence*

The vertices of $\triangle PQR$ are $P(2, 2)$, $Q(3, 4)$, and $R(5, 2)$. The notation $(x, y) \rightarrow (x + 1, y - 6)$ describes the translation of $\triangle PQR$ to $\triangle XYZ$. Show that $\triangle PQR \cong \triangle XYZ$ to verify that the translation is a congruence transformation.

Solution

S You can see that

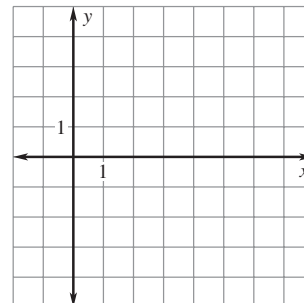
$PR = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$, so $\overline{PR} \cong \underline{\hspace{1cm}}$.

A Using the slopes, $\overline{PQ} \parallel \underline{\hspace{1cm}}$ and $\overline{QR} \parallel \underline{\hspace{1cm}}$. If you extend \overline{PQ} and \overline{XZ} to form $\angle V$, the Corresponding Angles Postulate gives you

$\underline{\hspace{1cm}} \cong \angle V$ and $\angle V \cong \underline{\hspace{1cm}}$. Then, $\underline{\hspace{1cm}} \cong \underline{\hspace{1cm}}$ by the Transitive Property of Congruence.

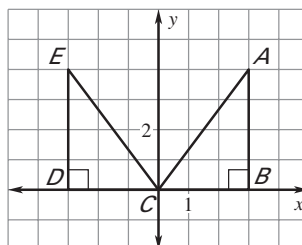
S Using the distance formula, $PQ = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ so $\overline{PQ} \cong \underline{\hspace{1cm}}$. So, $\triangle PQR \cong \triangle XYZ$ by the $\underline{\hspace{1cm}}$.

Because $\triangle PQR \cong \triangle XYZ$, the translation is a congruence transformation.



✓ Checkpoint Complete the following exercise.

5. Show that $\triangle ABC \cong \triangle EDC$ to verify that the transformation is a congruence transformation.



Homework